ELSEVIER

Cement and Concrete Composites 21 (1999) 29–37

Cement & Concrete Composites

# Determination of the dividing strength and its relation to the concrete strength in lightweight aggregate concrete

H.J. Chen*, T. Yen, T.P. Lia, Y.L. Huang

*Department of Civil Engineering, National Chung-Hsing University, Taichung 40227, Taiwan*

Received 4 September 1997; accepted 27 July 1998

## Abstract

As concrete is considered to be a composite material consisting of mortar and coarse aggregate, its strength depends on the mortar strength and coarse aggregate strength. During the strength development stage of a lightweight aggregate concrete, a critical condition, under which the type of stress distribution changes, occurs as the values of the modulus of elasticity of the lightweight aggregate and mortar become the same. The concrete strength corresponding to this instant is named 'Dividing Strength ($F_G$)' by Weigler & Karl (*Stahlleichtbeton*, 1972, pp. 38–43) [1]. A series of specimens consisting of 252 concrete cylinders ($100\phi \times 200$ mm) and 252 mortar cubes ($50 \times 50 \times 50$ mm) were made and subjected to compression tests. From the obtained mortar–concrete strength relationship, we were able to identify a dividing strength for a lightweight aggregate concrete made with a 0.4 water/cement ratio. The test data also show that both particle density and particle sizes of lightweight aggregate are governing factors for the dividing strength. The concept of a dividing strength can be utilized to optimize a mix design for lightweight aggregate concrete. For instance, concrete that has a designed compressive strength much greater than its dividing strength may consume too much cement and resulting to wasteful cement consumption.

*Keywords:* Lightweight aggregate; Lightweight aggregate concrete; Dividing strength

## 1. Introduction

The achievable strength of lightweight coarse aggregate concrete (LWAC) is generally determined by the individual strengths of the aggregate and mortar as well as the type of stress distribution, which is further affected by the relative values of the modulus of elasticity ($E$) for the aggregate and mortar [2,3]. Concrete is considered to be a composite material mixed from mortar and coarse aggregate. The former acts as a matrix of binder material and the latter behaves as supplementary material. For a normal weight concrete (NWC), because the $E$ value for the normal weight aggregate is higher than that of the mortar, the aggregate becomes the major supporting material. The mortar, on the other hand, can only be treated as a medium for stress transfer. As a result, fracture can always be detected in mortar during compression tests. In other words, the compressive load of the concrete is limited by the mortar strength. For LWAC, however, the aggregate becomes the minor supporting part since it normally possesses a smaller $E$ value. Fracture is most often found in the aggregate and the concrete strength is primarily controlled by the particle strength of the coarse lightweight aggregate [4,5].

To investigate how the aggregate strength affects the mechanical properties of lightweight aggregate concrete, many studies have been carried out [6–8]. By comparing the strength developments in mortar and lightweight aggregate concrete, Weigler & Karl [1] proposed a theory that there exists a 'dividing strength' ($F_G$), corresponding to a condition where a change of stress distribution occurs between the mortar and aggregate during the strength development of LWAC.

According to his concept, the relationship between the strengths of the concrete and mortar can be described in three ranges (see Fig. 1). The condition corresponding to the first range is similar to that of NWC; the strength of the concrete is determined by the mortar strength. Within the second range, as the modulus of elasticity of the coarse aggregate is lower than that of the mortar, concrete strength is restrained

---

*Corresponding author

0958-9465/99/$ — see front matter © 1999 Elsevier Science Ltd. All rights reserved.
PII: S0958-9465(98)00035-3

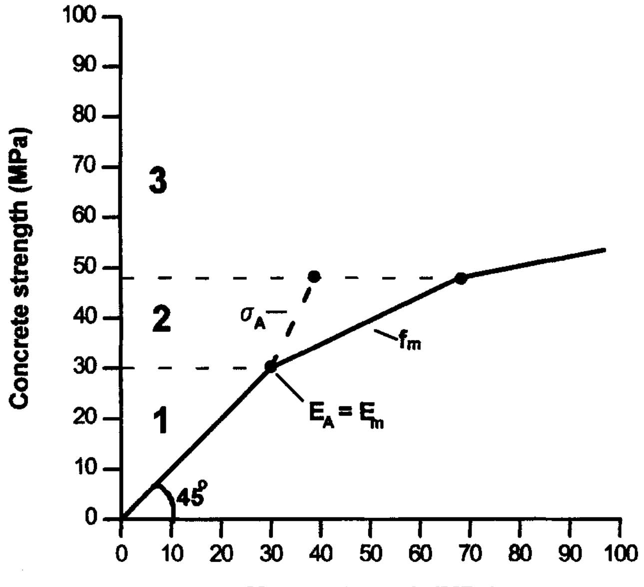

Fig. 1. Relationship between concrete strength and mortar strength [1].

by the strength of the coarse aggregate. Since two distinct tendencies of concrete strength are depicted in relation to mortar strength, it is implied that a change in the type of stress distribution occurs. The concrete strength corresponding to this turning point is called the dividing strength ($F_G$).

The goal of this experimental study is three-fold: first, to verify experimentally the existence of dividing strength in LWAC and to propose a method for evaluating the $F_G$ value. Second, to investigate how the particle density of LWA ($\rho_A$) and the maximum aggregate grain size ($D_{max}$) affect the $F_G$ value. Finally, to discuss the relationship between the $F_G$ value and the achievable strength of LWAC.

## 2. Test program

Three experimental procedures were carried out in the tests:

1. Verification of the existence of the dividing strength in LWAC by examining the relationship between the strength developments in LWAC and NWC with the same mix proportions at two different water/cement ratios (w/c = 0.40 and 0.78). The compression tests were conducted on specimens with nine curing ages (1, 2, 3, 4, 5, 7, 10, 14 and 28 days).
2. Evaluation of the $F_G$ value using regression techniques according to the relationship between the compressive strengths of the concrete and the mortar. The compression tests were conducted on specimens (cured for 28 days) with eight cement contents. (260, 279, 300, 325, 355, 390, 433 and 489 kg/m$^3$)
3. Investigation of the effect of $\rho_A$ and $D_{max}$ on the $F_G$ value of the lightweight aggregate concrete. The adopted coarse aggregate content and gradation followed ACI 211 [9] and ASTM C330 [10], respectively. Three groups of particle densities ($\rho_A$ = 500–800 kg/m$^3$, 1200–1400 kg/m$^3$ and 1500–1700 kg/m$^3$) and two $D_{max}$ values (19 and 12.5 mm) of LWA were selected.

### 2.1. *Materials*

Type 1 Portland cement, natural sand, normal weight aggregate, lightweight aggregate were used to cast mortar cubes and concrete cylinders. The selected cement has a specific gravity of 3.15 and fineness of 346 m$^2$/kg; the sand has a specific gravity of 2.62, a water absorption capacity of 1.3%, and a fineness modulus of 2.89; the normal weight aggregate has a specific gravity of 2.62 and a water absorption capacity of 1.1%. Two types of lightweight aggregates made from expanded shale in Taiwan and Mainland China were used. Aggregate made in China has particle density of 500–800 kg/m$^3$ and those made in Taiwan, 1200–1700 kg/m$^3$. Table 1 shows the particle sizes, particle density, and water absorption capacities of these materials.

Before the test materials were mixed, the fine and coarse normal weight aggregates were prepared under a saturated, surface-dry condition. The LWAs were stored under dry conditions. For the LWAC, an amount of water, corresponding to a 30-min absorption of LWA, was added for mixing in addition to the designed amount of water. The mix proportions are illustrated in Table 2.

### 2.2. *Test specimens*

Table 3 gives the detailed information on 252 concrete cylinders (100$\phi$ × 200 mm) and 252 mortar cubes (50 × 50 × 50 mm) that were made with mortar extracted from the fresh concrete. To maintain a mortar uniformity, mortar in concrete was filtered through #4 sieve (4.8 mm opening) for casting mortar cubes such that a compatibility between mortar and mortar in concrete can be realized. Conceivably, the shape and size effects on measured compressive strengths cannot be ignored. Since the results of the mortar strength test were considered to be a parameter for comparison, the size of the mortar specimens were selected as 50 × 50 × 50 mm cubes (different from the size of concrete specimens), which are commonly used in standard compression tests. The resulting concrete strength–mortar strength relationship, which will be discussed in the following sections, could vary as the

Table 1
Physical properties of lightweight aggregate

| Expanded shale of LWA | Grain size (mm) | Particle density (kg/m$^3$) | Water absorption at 30 min (%) | Water absorption at 24 h (%) | Relative density |
|---|---|---|---|---|---|
| $\rho_A$ = 500–800 kg/m$^3$ (made in China) | 25–19 | 530 | 11.5 | 20.2 | 2.63 |
| | 19–12.5 | 560 | 11.5 | 18.9 | 2.63 |
| | 12.5–9.5 | 650 | 11.1 | 16.6 | 2.63 |
| | 9.5–5 | 790 | 8.8 | 13.1 | 2.63 |
| $\rho_A$ = 1200–1400 kg/m$^3$ (made in Taiwan) | 25–19 | 1270 | 7.2 | 10.9 | 2.63 |
| | 19–12.5 | 1270 | 7.3 | 11.3 | 2.63 |
| | 12.5–9.5 | 1300 | 7.4 | 11.3 | 2.63 |
| | 9.5–5 | 1420 | 7.5 | 10.3 | 2.63 |
| $\rho_A$ = 1500–1700 kg/m$^3$ (made in Taiwan) | 25–19 | 1670 | 7.8 | 10.8 | 2.63 |
| | 19–12.5 | 1640 | 8.7 | 12.1 | 2.63 |
| | 12.5–9.5 | 1590 | 9.2 | 12.4 | 2.63 |
| | 9.5–5 | 1630 | 8.9 | 12.5 | 2.63 |

concrete and mortar specimens were made with different sizes. Qualitatively, however, the deficiencies were considered insignificant in the final conclusion to be drawn.

### 2.3. Compression tests

All of the test specimens were cured at a constant temperature (23°) and humidity (95%). The standard ASTM C109 and ASTM C39 compression test procedures were followed in testing cylinder and cube specimens respectively. Prior to the compression tests, specimens were removed from the curing room and left to dry in air for 24 h. Three mortar cubes or three concrete cylinders were tested successively to determine the average strengths for each group of specimens.

## 3. Test results and analysis

### 3.1. Verification of the existence of dividing strength

A lightweight aggregate concrete made with a high water/cement ratio, mortar strength generally develops a weak mortar. Consequently, the failure mode of such concrete would be similar to that of normal weight concrete. A low water/cement ratio concrete that has

Table 2
Mix proportions of lightweight aggregate concrete

$\rho_A$ = 1200–1400 kg/m$^3$

| $D_{max}$ (mm) | Cement (kg/m$^3$) | Slump (mm) | w/c | Water (kg/m$^3$) Content | Water (kg/m$^3$) Absorption of LWA (30 min) | LWA (kg/m$^3$) | Natural sand (kg/m$^3$) |
|---|---|---|---|---|---|---|---|
| 19.0 | 260 | 40–80 | 0.75 | 195 | 41 | 560 | 734 |
| | 279 | | 0.70 | | 41 | 556 | 727 |
| | 300 | | 0.65 | | 41 | 549 | 720 |
| | 325 | | 0.60 | | 40 | 544 | 712 |
| | 355 | | 0.55 | | 40 | 536 | 702 |
| | 390 | | 0.50 | | 39 | 528 | 690 |
| | 433 | | 0.45 | | 38 | 516 | 676 |
| | 489 | | 0.40 | | 37 | 502 | 658 |
| 12.5 | 260 | 40–80 | 0.80 | 207 | 38 | 518 | 799 |
| | 279 | | 0.74 | | 38 | 515 | 792 |
| | 300 | | 0.69 | | 38 | 510 | 781 |
| | 325 | | 0.64 | | 38 | 506 | 770 |
| | 355 | | 0.58 | | 37 | 499 | 756 |
| | 390 | | 0.53 | | 37 | 494 | 740 |
| | 433 | | 0.48 | | 36 | 485 | 720 |
| | 489 | | 0.42 | | 35 | 474 | 694 |

Table 3
The number of test specimens with different experimental procedures

| Test specimens | | $\rho_A = 500–800$ kg/m$^3$ | | $\rho_A = 1200–1400$ kg/m$^3$ | | $\rho_A = 1500–1700$ kg/m$^3$ | | NWA |
|---|---|---|---|---|---|---|---|---|
| | | $D_{max}$ = 19 mm | $D_{max}$ = 12.5 mm | $D_{max}$ = 19 mm | $D_{max}$ = 12.5 mm | $D_{max}$ = 19 mm | $D_{max}$ = 12.5 mm | $D_{max}$ = 19 mm |
| Strength development (nine curing ages) | Concrete (100$\phi$ × 200) | – | – | 9 × 3 = 27 | – | – | – | 9 × 3 = 27 |
| | | – | – | 9 × 3 = 27 | – | – | – | 9 × 3 = 27 |
| | Mortar (50 × 50 × 50) | – | – | 9 × 3 = 27 | – | – | – | 9 × 3 = 27 |
| | | – | – | 9 × 3 = 27 | – | – | – | 9 × 3 = 27 |
| Determining $F_G$ (eight cement contents) | Concrete (100$\phi$ × 200) | 8 × 3 = 24 | 8 × 3 = 24 | 8 × 3 = 24 | 8 × 3 = 24 | 8 × 3 = 24 | 8 × 3 = 24 | – |
| | Mortar (50 × 50 × 50) | 8 × 3 = 24 | 8 × 3 = 24 | 8 × 3 = 24 | 8 × 3 = 24 | 8 × 3 = 24 | 8 × 3 = 24 | – |
| Total: concrete specimens = 252, mortar specimens = 252 | | | | | | | | |

stronger mortar than aggregate generally fails first in aggregate during a compression test.

Fig. 2(a) compares the measured mortar strengths and concrete strengths ratio at various ages, up to 28 days. Fig. 2(b) compares the similar data for concrete made with a 0.78 water/cement ratio. In the case of the lower water/cement ratio [Fig. 2(a)], the mortar strength development curves for NWC and LWAC are rather similar. However, the corresponding trends for concrete strength development appear rather different. Presumably, since both the mortars in the NWC and LWAC specimens are almost identical (same water/cement ratio), the two mortar strength development curves should coincide. However during the mixing period, the LWA will absorb water since it had been prepared under a dry conditions. The deviation of the curves is considered due to the different extent of water absorption by the normal weight and lightweight aggregates. The strength differentials between mortar and concrete for lightweight aggregate almost double that of normal weight aggregate, particularly for strengths measured on or after 7 days of curing. This suggests that lightweight aggregate is the possible culprit causing failure to concrete under loading and the existence of the so-called dividing strength.

The same data used to construct Fig. 2(a) were utilized to establish a strength relationship between mortar and concrete with respect to specimen ages, as shown in Fig. 3. A point of deflection can be clearly observed on the curve representing the lightweight aggregate, but not the normal weight aggregate. Understandably, the early strength development is controlled by the mortar strength throughout the first segment of the curve. It then increases with a slower rate in concrete and forms a flatter second segment. The point of deflection reflects the existence of a dividing strength of a lightweight aggregate concrete tested.

Figure 3 has the first segment with a slope less than 45° demonstrated by the Weigler idealized model (Fig. 1). The reason for the deviation may be associated with the shape/size effects and possible bond failure along the interface between mortar and aggregate. Since the values for $F_G$ were regarded as an index, qualitatively, the deficiencies were supposed insignificant in the conclusions to be drawn.

Interestingly, no point of deflection can be depicted for the lightweight aggregate concrete made with a 0.78 water/cement ratio and normal weight concrete as shown in Fig. 2(b). This means no dividing strength was reached and failures took place in the mortar rather than through aggregate particles during the compression test.

### 3.2. Determination of dividing strength

A series of mortar cubes and concrete cylinders made with eight different cement contents as shown in Table 2 were tested to establish the strength relationship between mortar and lightweight aggregate concrete. Figure 4(a–c) illustrates these relationships for concrete made with aggregates having $\rho_A = 1500–1700$, 1200–1400 and 500–800 kg/m$^3$, respectively. The strength data were divided into two groups, upon each group a linear regression procedure was carried out until the computed residues became small enough to be neglected. The intersection of the two regressed lines determines the dividing strength.

For example, the aggregate particle with $D_{max} = 19$ mm, the five data points in the first segment appear to be on a straight line. A linear regression for these data points yields a regressed line, $L_1$. The computed correlation coefficient, $R^2 = 0.993$, is statistically representative and mathematically acceptable. Likewise, a linear regressed line, $L_2$, for the four data points in the second segment can be established with a correlation coefficient, $R^2 = 0.989$. The intersection point of $L_1$ and $L_2$ yields a dividing strength of 35.3 MPa. Similarly, dividing strengths for $D_{max} = 12.5$ mm and the other aggregates of different particle densities were determined as shown in Fig. 4(a–c). The computed correlation coefficients are almost 1.0 as shown in Fig. 5(a).

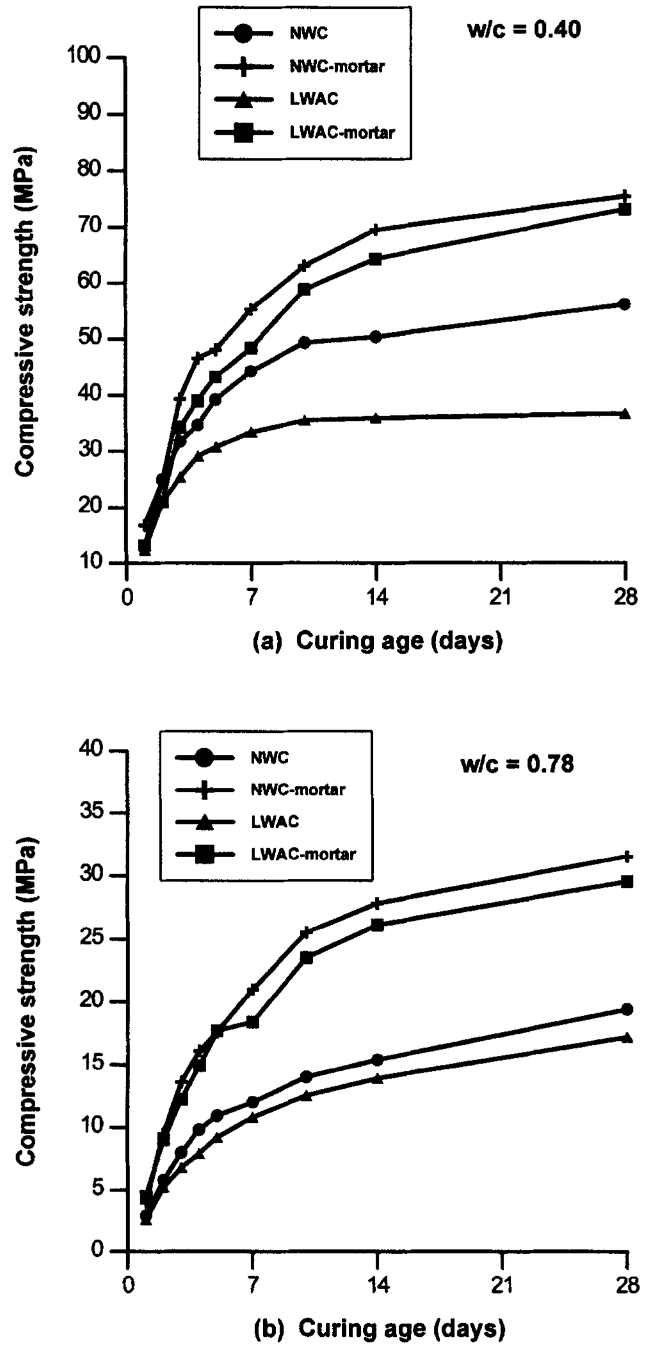

Fig. 2. Strength development of concrete and mortar.

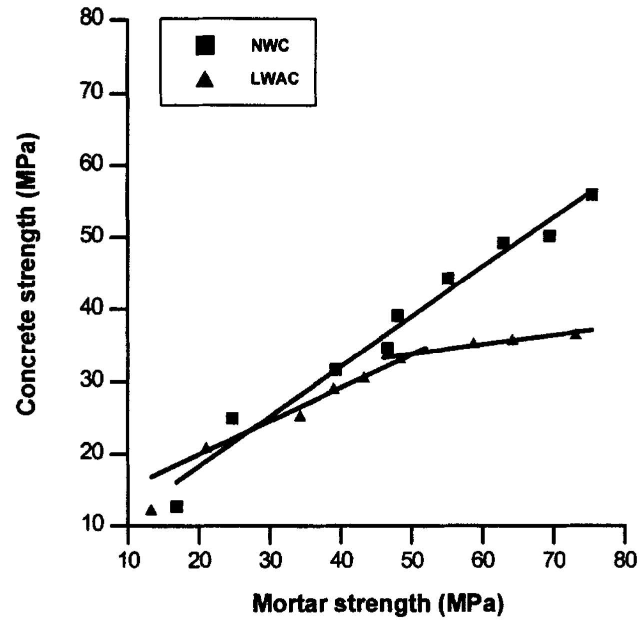

Fig. 3. Relationship between the strength of concrete and mortar.

The results from Fig. 5(b) show that the slopes of the regressed line $L_2$ are rather mild compared to those of $L_1$. In other words, the resulting $F_G$ value does not appear very sensitive to the selection of data groups. By using the proposed procedure, therefore, the deviation of $F_G$ is rather limited.

With a prescribed proportion (see Table 2), different cement contents would result in different contents for fine and coarse aggregates. For example, the aggregate particle with $D_{max} = 19$ mm, the LWA contents are in a range of 502–560 kg/m$^3$. To be consistent, supplemental experiments were conducted to control the contents of the coarse aggregate in the concrete as well as the sand in the mortar subjected to a certain cement content. For example, in the aggregate particle with $D_{max} = 19$ mm, the LWA content is equal to 540 kg/m$^3$ (mix proportions shown in Table 4). In the experiments, to determine the dividing strength, also using the relationship between the compressive strength of the concrete and the mortar, whereas the values for compressive strength were taken at various curing ages(1, 2, 3, 5, 7, 10, 14, 28 and 56 days) within the same batch of concrete. Figure 6 shows that although the methods used to obtain the dividing strength are different, the relative difference is within about 6.5%, and the data shows the same trend of change following $D_{max}$ or coarse aggregate content change, revealing that these strength values can be reasonably considered as an indicator to quantify the dividing strength of the lightweight aggregate concrete.

### *3.3. Effect of particle density*

According to Weigler & Karl's suggestion[1], the $F_G$ value is primarily controlled by the particle strength of the aggregate. As the particle strength of the coarse aggregate is highly influenced by the particle density, it implies that the $F_G$ value is closely related to the particle density of the coarse aggregate.

To assess aggregate strength is a rather time consuming procedure and results may not be as accurate as testing regular cylinders or cubes. To measure particle density is rather simple. For this reason, the particle density may be a convenient variable for determining the dividing strength. Figure 7 summarizes the relationships between dividing strength and three different particle densities, $\rho_A$, and two aggregate sizes of $D_{max} = 19$ and 12.5 mm. The group with $\rho_A = 1500$–$1700\ kg/m^3$ developed the highest dividing strength, and was followed by $\rho_A = 1200$–$1400$ and 500–800 $kg/m^3$ groups successively. This indicates

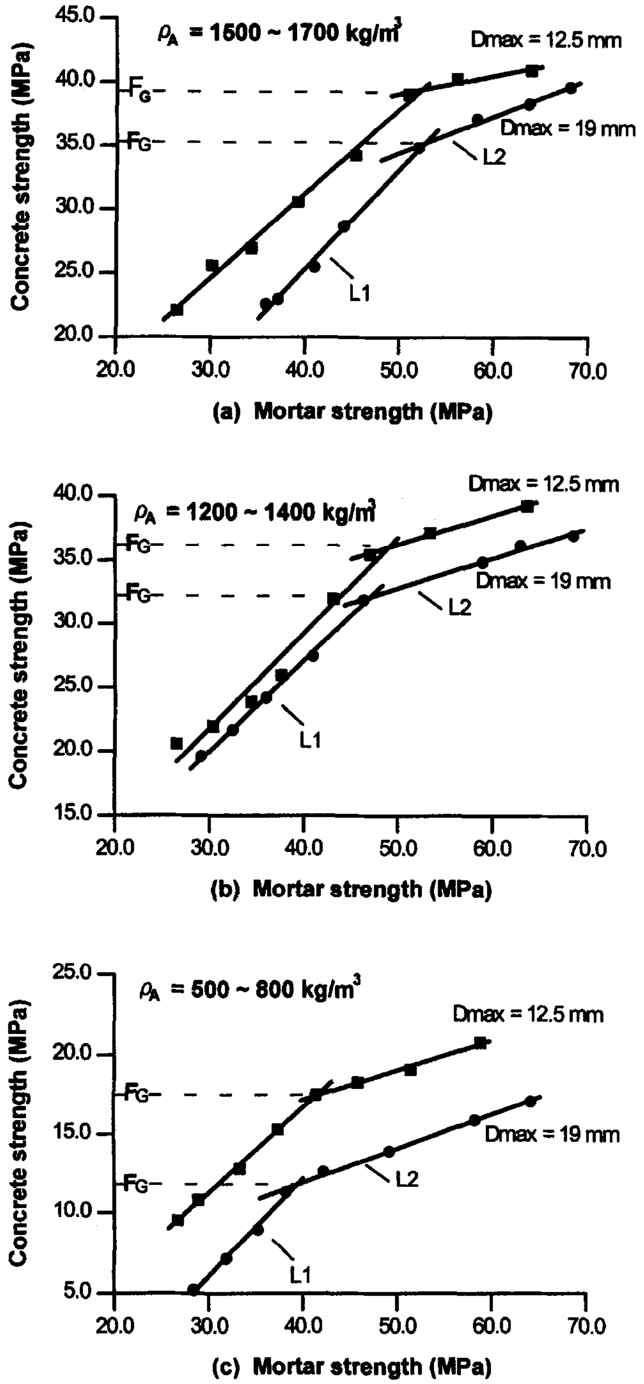

Fig. 4. Determination of dividing strength by regression.

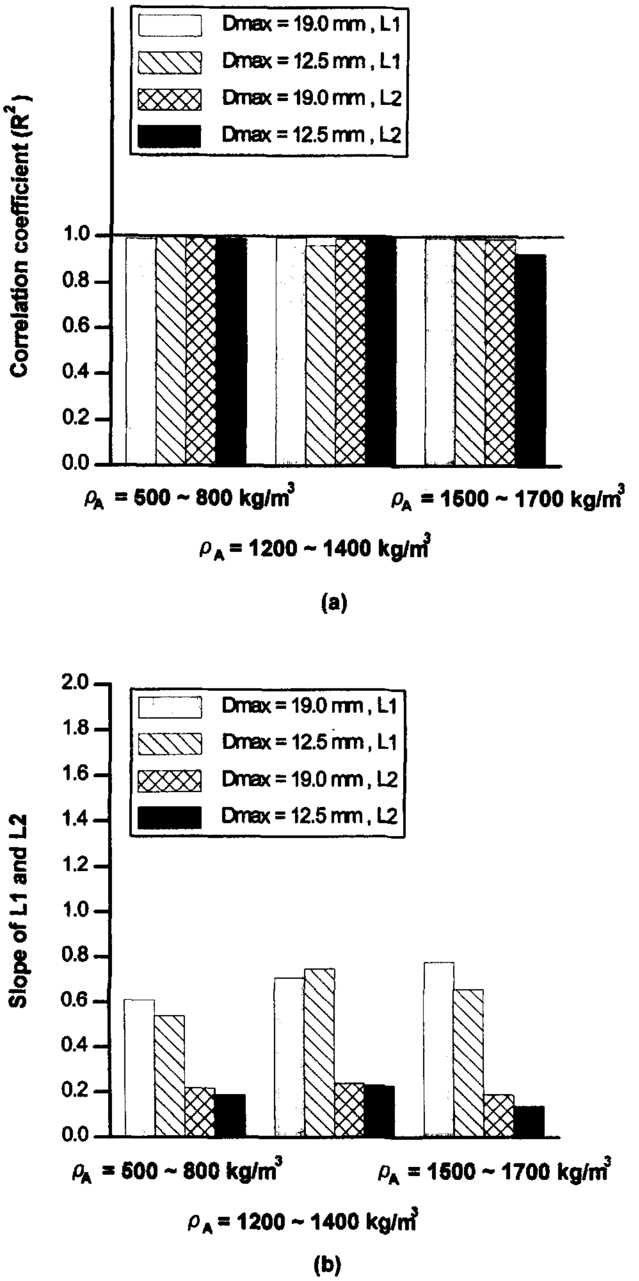

Fig. 5. Regression results of $R^2$ and line slope.

Table 4
Mix proportions of supplemental tests

| $\rho_A$ = 1200–1400 kg/m$^3$ | | | | | |
|---|---|---|---|---|---|
| $D_{max}$ (mm) | Cement (kg/m$^3$) | LWA content (kg/m$^3$) | Water (kg/m$^3$) | | Natural sand (kg/m$^3$) |
| | | | Content | Absorption of LWA (30 min) | |
| 19 | 415 | 540 | 195 | 41 | 679 |
| 12.5 | 438 | 487 | 206 | 37 | 743 |
| 9.5 | 457 | 412 | 215 | 31 | 805 |

that lower particle density of the aggregate results in lower dividing strength.

### 3.4. *Effect of maximum aggregate size*

Physically, as the lightweight aggregate is produced with the same production procedure, a smaller grain size will generally lead to a larger particle strength. Additional tests on lightweight aggregate particles show that the average ultimate point-load decreases as grain size increases (see Fig. 8). Since the ultimate point-load of particle is closely related to the particle strength, a decrease of $D_{max}$ should tend to promote an increase in the $F_G$ value. This feature is clearly shown in the results in Fig. 7. In all three $\rho_A$ groups, the $F_G$ values for $D_{max}$ = 12.5 mm are greater than those for $D_{max}$ = 19 mm. As $D_{max}$ increases, particle strength decreases. The mortar strength is prone to surpass the aggregate strength, causing an early appearance (a lower value) of $F_G$.

### 3.5. *Dividing strength and achievable strength*

Generally, the compressive strength of LWAC increases as the cement content increases. However, experimental results indicate that after reaching a certain compressive strength, the tendency for the increase of compressive strength becomes milder and then the compressive strength is determined by $F_G$. For instance, Fig. 9(a) shows that for LWA with $\rho$A = 1500–1700 kg/m$^3$ and $D_{max}$ = 19 mm, the growth

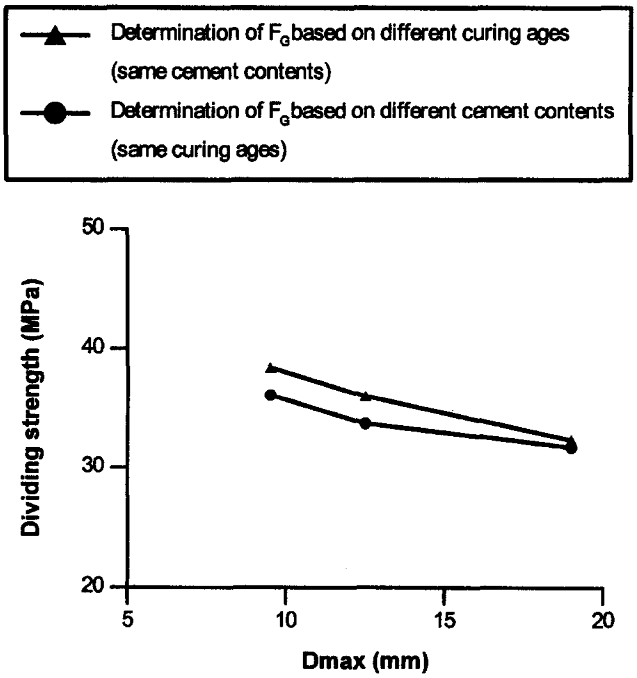

Fig. 6. Comparison of the dividing strength.

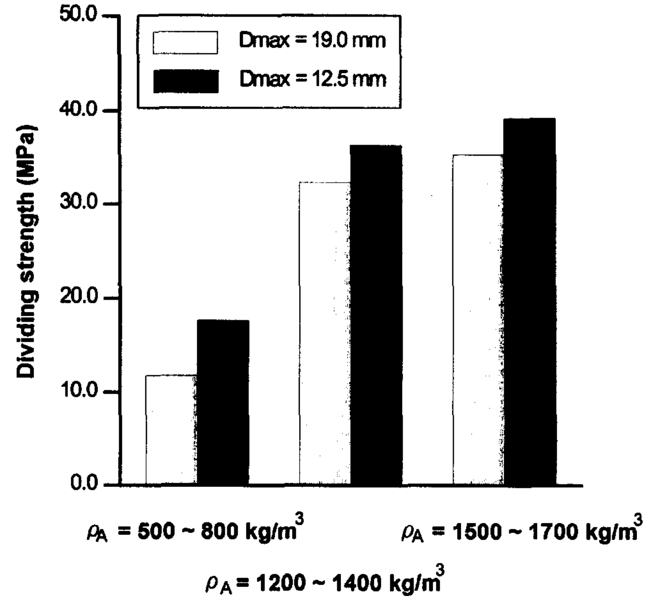

Fig. 7. Dividing strength for various LWA.

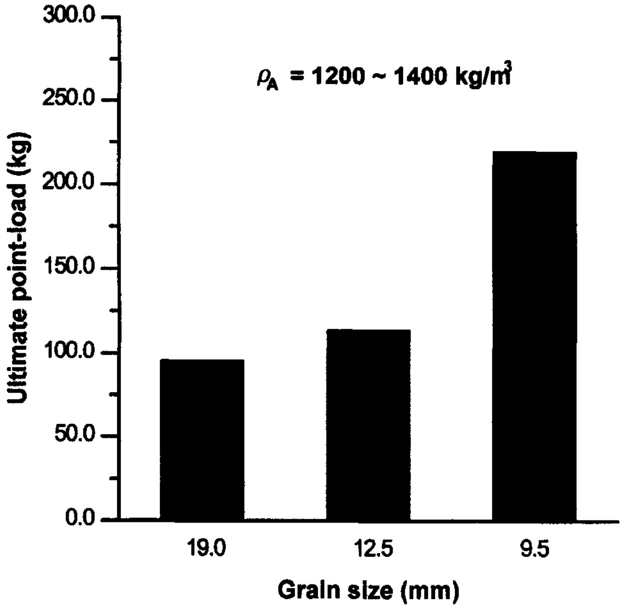

Fig. 8. Ultimate point loading of LWA.

rate of the compressive strength for LWAC reduces when the compressive strength exceeds the FG value (similar results are also found for the other two groups of $\rho_A$). Additionally, the results from Fig. 9 also show that for LWAC with same cement content, the higher the $F_G$ value, the higher the concrete strength. If the data corresponding to a certain value for $\rho_A$ and $D_{max}$ in Fig. 9 are represented as two sectional regressed lines, separation based on the corresponding location of $F_G$, the additional achievable strengths for the LWAC per unit increment of cement content (the slope of the regression line) can be evaluated, as shown in Fig. 10. When the concrete strength is lower than $F_G$, the additional achievable strength is within the range of about 0.09–0.15 MPa/kg. When the concrete strength is higher than $F_G$, the range is from 0.02 to 0.04 MPa/kg. From the viewpoint of the LWAC proportion design, it is important to point out that as the designed concrete strength is greater than $F_G$, the beneficial achievable strength per unit increment of cement content is considered uneconomical.

## 4. Conclusion

Several conclusions can be drawn as follows:

1. Both the normal weight concrete and normal weight mortar had a similar strength development pattern. The strength development for a lightweight aggregate concrete made with a low water/cement ratio may be governed by its aggregate strength. It was found that a dividing strength exists in lightweight aggregate concrete of a low water/cement ratio.
2. The strength development of a lightweight aggregate concrete made with a low water/cement ratio develops a distinct two-segment relationship between concrete strength and mortar strength. By applying the linear regression technique, the dividing strength can be assessed by the point of intersection of the two regressed line segments.
3. A heavy particle density of lightweight aggregate produces a strong dividing strength.

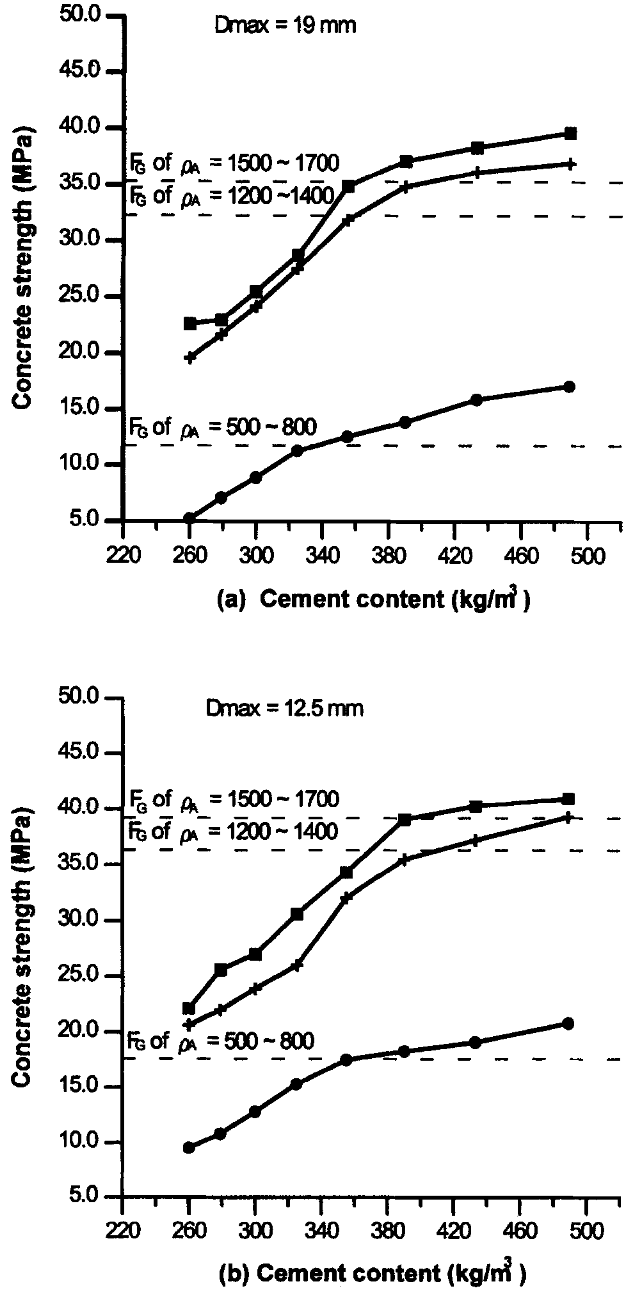

Fig. 9. LWAC strength in relation to cement content.

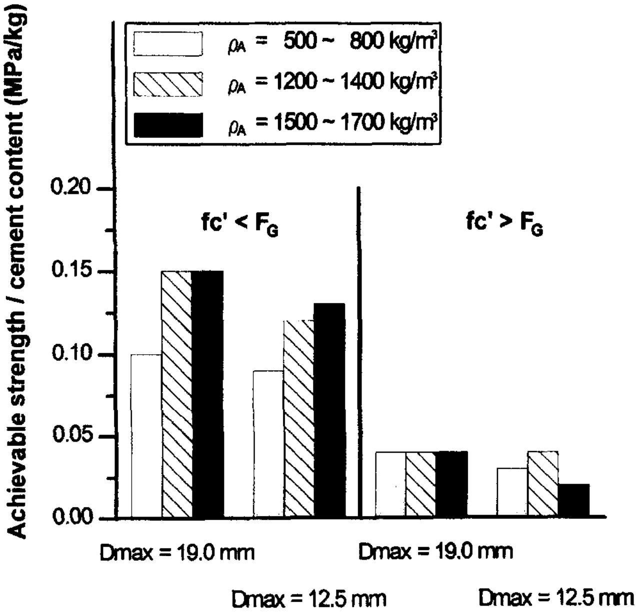

Fig. 10. Achievable strength per unit increment of cement content.

4. The smaller aggregate sizes yield greater dividing strength. This is particularly true for a lightweight aggregate concrete of a low water/cement ratio.
5. A high dividing strength is generally attributed to a high achievable concrete strength.
6. From the viewpoint of LWAC proportion design, as the designed concrete strength is greater than dividing strength, the beneficial achievable strength per unit increment of cement content is considered uneconomical.

## References

[1] Weigler H, Karl S. Stahlleichtbeton. Bauverlag GMBH, Wiesbaden and Berlin, 1972, pp. 38–43.

[2] Wischers G. Aufnahme von Druckkraften in Schwerbeton und in Leichtbeton. Beton 1967;5: 184

[3] Zhang M-H, Gjorv OE. Characteristics of lightweight aggregates for high-strength concrete. ACI Materials Journal 1991;Mar–Apr:150–158.

[4] Short A, Kinniburgh W. Lightweight Concrete. MCMLXIII, London, 1963, pp. 1–14.

[5] Chen HJ, Yen T, Ko CT. Influences of properties and gradation of lightweight aggregate on the $F_G$ of lightweight aggregate. In International Symposium on Structural Lightweight Aggregate Concrete, Norway, June 1995, pp. 472–480.

[6] Hammer TA, Smeplass S. The influence of lightweight aggregate properties on material properties of the concrete. In *International Symposium on Structural Lightweight Aggregate Concrete*, Norway, June 1995, pp. 517–532.

[7] Zhang M-H, Gjorv OE. Mechanical properties of high-strength lightweight concrete. ACI Materials Journal 1991;Mar–Apr: 240–247.

[8] Slate FO, Nilson AH, Martinez S. Mechanical properties of high-strength lightweight concrete. ACI Journal 1986;Jul–Aug:606–613.

[9] ACI Standard. Standard Practice for Selecting Proportions for Structural Lightweight Concrete (ACI.211.2–81).

[10] Annual ASTM Standard, Vol. 04.01–04.02, 1993.